A FAST ANTI-NOISE FUZZY C-MEANS ALGORITHM FOR IMAGE SEGMENTATION

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ABSTRACT
Conventional fuzzy C-means (FCM) algorithm does not consider spatial information in the clustering, which makes it sensitive to noise and inefficient. In order to overcome these problems, we propose a fast anti-noise FCM algorithm for image segmentation, which constructs a new spatial function by combining pixel gray value similarity and membership. This spatial function is used to update the membership which in turn is used to obtain the cluster centers iteratively. The proposed algorithm can achieve desirable segmentation results in less iterations and reduce the effect of noise effectively. Experimental results show that the proposed algorithm outperforms conventional FCM and other extended FCM algorithms.

Index Terms— Image segmentation, fuzzy clustering, fuzzy C-means, spatial information.

1. INTRODUCTION
Fuzzy C-means (FCM) algorithm is a popular algorithm and has been widely applied in computer vision and image processing. It allows each pixel to belong to multiple clusters with certain degree, which offers robust characteristics for ambiguity and retain more information than hard segmentation methods [1]. However, conventional FCM algorithm does not consider any spatial information, which makes it perform poorly in low contrast, inhomogeneity and noisy images [2]. In order to overcome these problems, many modified FCM algorithms with spatial information incorporated have been proposed. Commonly, these modified algorithms utilize spatial information mainly through modifying the objective function [3, 4, 5, 6] or altering the distance measurement between pixels and cluster centers [7].

Chuang et al. [8] proposed an improved FCM (SAFCM) algorithm incorporating the spatial information, and the membership values are altered after the cluster distribution in the neighborhood has been considered. Ahmed et al. [9] proposed FCM algorithm where the objective function is modified by introducing an neighborhood term, which is used to compensate the intensity inhomogeneity and allow the labeling of a pixel to be influenced by the labels of its immediate neighborhood. However, FCM algorithm is very time-consuming [10], since it needs to compute the neighborhood labeling during each iteration. A novel robust fuzzy local information C-means clustering (FLICM) algorithm [11] with local spatial and gray level information incorporated is free of the empirically adjusted parameters and enhances the clustering performance.

An enhanced FCM (EnFCM) algorithm [12] was proposed to speed up the clustering process. It uses a linearly-weighted sum image formed from both original image and each pixel’s local neighborhood average gray level. In addition, clustering is performed on the basis of the gray level histogram instead of pixels of the summed image. Hence, the computational time of EnFCM algorithm is reduced greatly since the number of gray levels in an image is much smaller than that of pixels. Fast generalized FCM (FGFCM) algorithm [13] introduces the spatial information combining the intensity of the local pixel neighborhood and the number of gray levels in an image. The quality of segmentation result is well enhanced, and the computational time of FGFCM algorithm is small because clustering is performed on the basis of the gray level histogram.

Although the above modified FCM algorithms have improved the performance of conventional FCM algorithm to some extent, they still have the following disadvantages: (1) they are sensitive to noise and outliers; (2) they require a large number of iterations to achieve the convergence of the iterations.

By analyzing these improved algorithms, this paper presents a fast and robust FCM (FRFCM) algorithm for image segmentation in order to overcome the problems mentioned above. FRFCM algorithm introduces a spatial function by combining pixel gray value similarity and membership to update the membership in each iteration. During each iteration, FRFCM firstly calculates each pixel’s membership based on the objective function incorporating spatial information of FCM, then it updates the membership using the newly-
constructed spatial function, and the updated membership function is used to calculate the cluster centers and objective function. In subsequent section we will see that FRFCM algorithm can achieve satisfying results in less iterations, providing great robustness to noise.

2. CONCURRENT FCM ALGORITHMS

Conventional FCM algorithm is an iterative optimization. It is based on minimizing the following objective function [14]:

$$E_{FCM} = \sum_{i=1}^{C} \sum_{j=1}^{N} \mu_{ij}^v \|x_j - v_i\|^2$$  \hspace{1cm} (1)

where $C$ is the number of clusters, $N$ is the number of pixels, $m$ is the weighting exponent that controls the fuzziness of the resulting partition, $x_j$ is the $j$th pixel in the image, $v_i$ is the center of the $i$th cluster and $\mu_{ij}$ represents the membership of $x_j$ belonging to $v_i$. The objective function is minimized under the following constraints:

$$\mu_{ij} \in [0, 1], \sum_{i=1}^{C} \mu_{ij} = 1$$  \hspace{1cm} (2)

As is seen, the formula (1) does not include any spatial information, it cannot achieve satisfying results when processing the low contrast, in-homogeneity and noisy images. Also, it performs poor in computation. To overcome these disadvantages, Ahmed et al. [9] proposed FCM-S where the formula (1) is modified in order to compensate the intensity inhomogeneity and allow the labeling of the pixel to be influenced by the labels of its immediate neighborhood. The objective function $E$ of FCM-S is defined as follows:

$$E_{FCM-S} = \sum_{i=1}^{C} \sum_{j=1}^{N} \mu_{ij}^v (\|x_j - v_i\|^2 + \frac{\alpha}{N_{R_{ij}} x_j} \sum_{r_j \in N_{ij}} \|x_r - v_i\|^2)$$  \hspace{1cm} (3)

where $N_{ij}$ represents the set of the neighboring pixels of $x_j$, and $N_{R_{ij}}$ is the cardinality of $N_{ij}$, $x_r$ is the pixel in $N_{ij}$, and $\alpha$ is one parameter to control the effect of the neighborhood term. As the probability that neighboring pixels with similar feature values belong to the same cluster is great, SAFCM algorithm [8] incorporated spatial information into the membership function for clustering. The spatial function is defined as:

$$h_{ij} = \sum_{x_r \in N_{ij}} \mu_{ir}$$  \hspace{1cm} (4)

where $\mu_{ir}$ represents the membership of pixel $x_r$ in the neighborhood $N_r$ of $x_j$ belonged to the $i$th cluster $v_i$.

Although FCM-S and SAFCM obtained good image segmentation results on a series of images, they do not have insufficient robustness to noise and the speed of achieving convergence is not fast.

3. FAST ANTI-NOISE FCM ALGORITHM

One of the most important characteristics of an image is that neighboring pixels are commonly highly-correlated, that is to say, neighboring pixels have similar feature values and the probability that they belong to the same cluster is great. The effective utilization of spatial information has great significance in improving the effectiveness and efficiency of the algorithm.

FCM-S and SAFCM utilize spatial information in different ways, yet they still lack enough robustness to noise. Besides, due to performing many iterations to achieve convergence, the two algorithms have low computational efficiency. Based on the analysis of the above problems, this paper presents a fast anti-noise FCM (FRFCM) algorithm for image segmentation.

FRFCM makes some modification to formula (3) as its objective function defined as follows:

$$E_{FRFCM} = \sum_{i=1}^{C} \sum_{j=1}^{N} \mu_{ij}^v (\|x_j - v_i\|^2 + \frac{\alpha}{N_{R_{ij}} x_j} \sum_{r_j \in N_{ij}} \|x_r - v_i\|^2)$$  \hspace{1cm} (5)

The membership values are updated according to the following formula (6) for $E_{FRFCM}$ to be minimized:

$$\mu_{ij} = \frac{(\|x_j - v_i\|^2 + \frac{\alpha}{N_{R_{ij}} x_j} \sum_{r_j \in N_{ij}} \|x_r - v_i\|^2)^{1/m}}{\sum_{k=1}^{C} (\|x_j - v_k\|^2 + \frac{\alpha}{N_{R_{ij}} x_j} \sum_{r_j \in N_{ij}} \|x_r - v_k\|^2)^{1/m}}$$  \hspace{1cm} (6)

The spatial function (4) only considers the membership of neighboring pixels, which results in small change of the membership of pixels. Thus, the algorithm needs to perform many iterations to achieve convergence and is relatively sensitive to noise. In order to overcome the problems mentioned above, FRFCM introduces a new spatial function by incorporating the gray similarity of neighboring pixel, formalized as $\|x_r - x_j\|^2$. The spatial information incorporated not only makes FRFCM insensitive to noise effectively; but also makes membership values change greatly in each iteration, quickly approaching the ideal values, and in turn makes FRFCM achieve satisfying segmentation results in less iterations. The new spatial function is defined as follows:

$$\omega_{ij} = \sum_{x_r \in N_{ij}} \mu_{ir} \|x_r - x_j\|^2$$  \hspace{1cm} (7)

where $\omega_{ij}$ represents the probability that pixel $x_j$ belongs to cluster $v_i$, and in turn it is incorporated to a new membership function (8) [8] to update the membership values.

$$\mu_{ij} = \frac{\mu_{ij}^{v_i} \omega_{ij}^{v_i}}{\sum_{k=1}^{C} \mu_{ij}^{v_i} \omega_{ij}^{v_i}}$$  \hspace{1cm} (8)

where $p$ and $q$ are parameters to control the importance of the spatial function $\omega_{ij}$ and the membership function $\mu_{ij}$.
The cluster centers are updated according to the following formula (9) with the updated membership values $\mu_{ij}$ incorporated.

$$\nu_i = \sqrt{\frac{\sum_{j=1}^{N} \mu_{ij}^m (x_j^2 + \frac{\alpha}{N} \sum_{x_r \in N_j} x_r^2)}{(1 + \alpha) \sum_{k=1}^{C} \mu_{ik}^m}}$$ (9)

During each iteration, FRFCM firstly calculates each pixel’s membership $\mu_{ij}$ according to (6), then it calculates the updated membership $\mu_{ij}'$ according to (7) and (8), and calculates the cluster centers according to (9).

### 4. EXPERIMENTAL RESULTS

In this section, we will present the performance of SAFCM, FCM$_S$, EnFCM, FGFCM and our FRFCM on a series of images including noisy synthetic images and medical images. We did not present the segmentation results of FCM due to poor results.

Fig. 1 and Fig. 2 show the results of applying SAFCM, FCM$_S$, EnFCM, FGFCM and FRFCM on synthetic noisy images. The results in Fig. 1(b)-1(e) and Fig. 2(b)-2(e) show that SAFCM, FCM$_S$, EnFCM and FGFCM are influenced by noise and cannot achieve satisfying results, while Fig. 1(f) and Fig. 2(f) show that FRFCM have strong robustness to noise like other image denoising algorithms [15, 16], it can achieve satisfying results and performs better than the other four algorithms.

![Fig. 1. Comparison of the segmentation results on a synthetic image corrupted by Gaussian noise (0 mean, 0.01 variance).](image1)

![Fig. 2. Comparison of the segmentation results on a very noisy image.](image2)

Fig. 3, Fig. 4 and Fig. 5 present comparison of segmentation results between SAFCM, FCM$_S$, EnFCM, FGFCM and FRFCM, when applied on MR images with intensity inhomogeneity and noise. The marked four regions with red rectangles in Fig. 3(a), three regions with red and green rectangles in Fig. 4(a) and three regions with red rectangles in Fig. 5(a) have great difference in the segmentation results. Through comparing the corresponding regions in Fig. 3(b)-3(f), Fig. 4(b)-4(f) and Fig. 5(b)-4(f) of the marked regions in Fig. 3(a), Fig. 4(a) and Fig. 5(a), we can obviously see that FRFCM is more insensitive to noise and can retain more original information.

![Fig. 3. Comparison of the segmentation results on an MR image.](image3)

Furthermore, comparison results in terms of the number of iterations and the running time (in seconds) until the final segmentation (i.e., until the algorithms have converged or reach the maximum number of the iterations which was
Fig. 4. Comparison of the segmentation results on an MR image.

Table 1. Comparison of computation efficiency

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<th>CPU time (in seconds)</th>
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5. CONCLUSION

In this paper, we present a fast anti-noise FCM (FRFCM) algorithm that constructing a new spatial function incorporating pixel gray value similarity and membership, which makes the algorithm reduce the effect of noise effectively and achieve ideal results in less iterations. Experimental results of our proposed algorithm and other four algorithms SAFCM, FCM_S, EnFCM and FGFCM illustrate that our proposed algorithm is more robust to noise and outperforms conventional FCM and other FCM extension algorithms.

Despite the improved performance of the proposed algorithm, there is still room for further improvement. In our current implementation, the parameters that control the impact of the spatial function and the membership function are set empirically. So our next major research issue is to explore the possibility of automatic and adaptive choosing optimal parameters in our algorithm, which will enhance the clustering performance in turn.

6. REFERENCES


